The Log-Periodic V Array

Here's a challenge — something to stimulate the investigative instincts of the serious antenna enthusiast.

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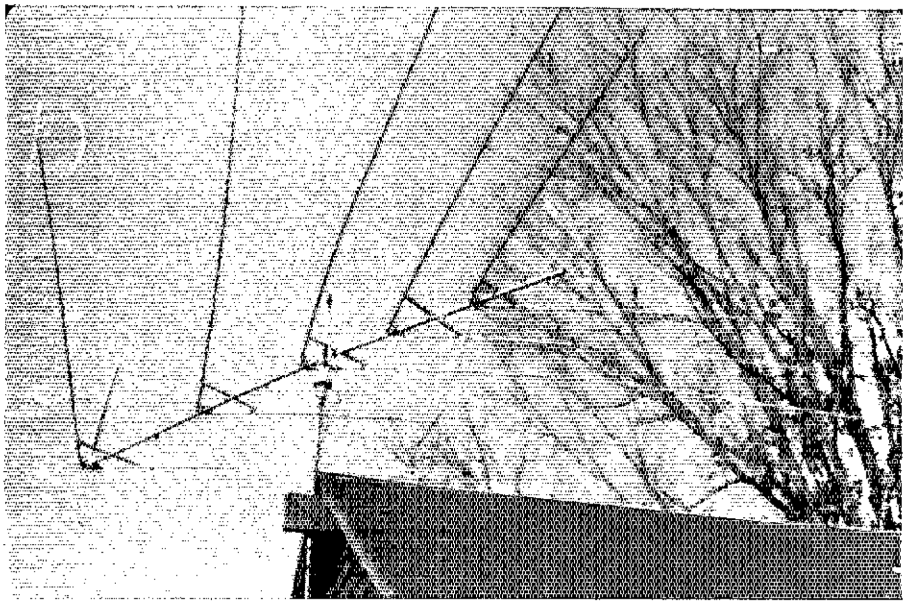
The following presentation is designed to familiarize the radio amateur with another concept in antenna-array design using driven elements. In two earlier works, it has been shown that the log-periodic dipole array (LPD) is a useful, unidirectional, high-gain (7 to 10 dBd), frequency-independent radiator.[1,2,3] The log-periodic resonant V array (LPV) is a modification of the LPD array as shown in (Fig. 1). Dr. Paul E. Mayes and Dr. Robert L. Carrel at the antenna laboratory of the University of Illinois, found that by simply tilting the elements toward the apex, the array could be operated in higher resonance modes with an increase in gain (9 to 13 dBd) and a pattern with negligible side lobes (Fig. 2).[4]

A higher resonance mode is defined as a frequency that is an odd multiple of the fundamental array frequency. For example, the higher resonance modes of 7 MHz are 21 MHz, 35 MHz, 49 MHz and so on. The fundamental mode is called the $\lambda/2$ (half-wavelength) mode, and each odd multiple as follows: $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, . . . etc., to the $(2n-1)\ \lambda/2$ mode. The usefulness of such an array becomes obvious when one considers an LPV with a fundamental-frequency design of 7 to 14 MHz that can also operate in the $3\lambda/2$ mode at 21 to 42 MHz. A four-band array can easily be developed yielding 7 dBd gain at 7 and 14 MHz and 10 dBd gain at 21 and 28 MHz, without traps. Also, using proper design parameters, the same array can be employed in the $5\lambda/2$ mode to cover the 35- to 70-MHz band. The 7- to 30-MHz LPV in use at K4EWG has been in service for one year and has performed well. However, minimum design



A pedestrian's view of the five-element 7- to 30-MHz log-periodic V showing one of the capacitive hats on the rearmost element.

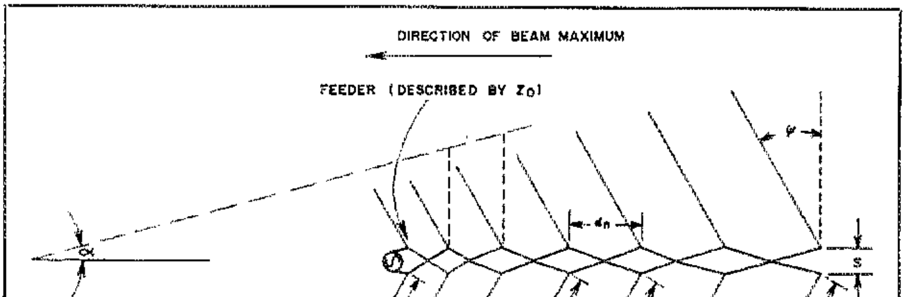


Fig. 1 — LPV schematic diagram and definition of terms.

[1]References appear on page 44.
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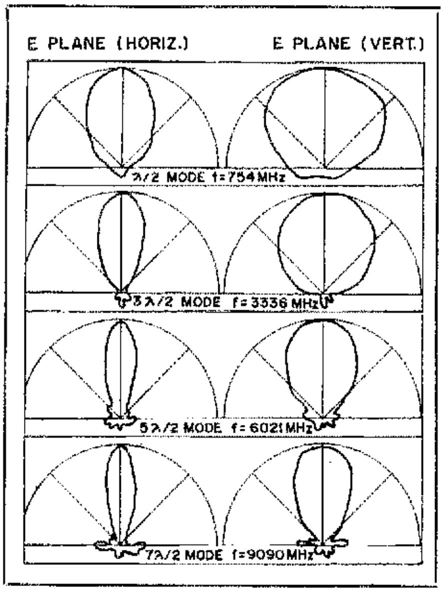


Fig. 2 — Typical radiation patterns from the Mayes and Carrel LPV in several modes. $\tau = 0.95$, $\sigma = 0.0268$, $\psi = 45°$ and $N = 25$.

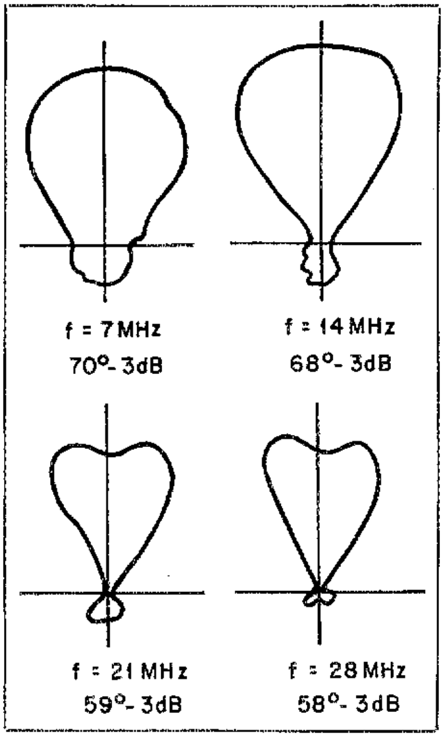


Fig. 3 — Horizontal radiation patterns of the K4EWG LPV array.

parameters were used and it is my opinion that a more conservative design (two additional elements) would yield a narrower half-power (3 dB) beamwidth on 40 and 20 meters. The author tested the LPV theory under the most extreme minimum design parameters (fewest elements and shortest boom), and the results confirmed the theory (Fig. 3).

Theory of Operation

The basic concepts of the LPD array also apply to the LPV array. The *Principle of Similitude*[5] is used in the LPD array design. That is, a series of interconnected "cells" or elements are constructed so that each adjacent cell or element differs by the design or scaling factor, τ (Fig. 4). That is, if ℓ_1 is the length of the longest element in the array and ℓ_n the length of the shortest, the relationship to adjacent elements is as follows:

$$\ell_1 = \frac{492}{f_1} \qquad \text{(Eq. 1)}$$

$\ell_2 = \tau\ell_1$
$\ell_3 = \tau\ell_2$
$\ell_4 = \tau\ell_3$, etc.
and, $\ell_n = \tau\ell_{n-1}$ (Eq. 2)

where
f_1 = lowest desired frequency and
n = total number of elements

Assume d_{12} is the spacing between elements ℓ_1 and ℓ_2. Then $d_{n-1,\,n}$ is the spacing between the last or shortest elements ℓ_{n-1} and ℓ_n, where n is equal to the total number of elements. The relationship to adjacent element spacings is as follows:

$d_{12} = 1/2\,(\ell_1 - \ell_2)\cot\alpha$
$d_{23} = \tau d_{12}$
$d_{34} = \tau d_{23}$
$d_{45} = \tau d_{34}$
. .
. .
. .

$$d_{n-1,\,n} = \tau d_{n-2,\,n-1} \qquad \text{(Eq. 3)}$$

where
α = 1/2 apex angle in degrees

It becomes obvious from an examination of the mathematical model that the elements, cells of elements and their associated spacings, differ by the design parameter τ. Each band of frequencies between any f and τf corresponds to one period of the structure. In order to be frequency independent (or nearly so), the variation in performance (impedance, gain, front-to-back ratio, pattern, etc.) across a frequency period must be negligible.

The "active region" is defined as the radiating portion or "cell" within the array which is being excited at a given frequency, f, within the array passband. As the frequency decreases, the active cell moves toward the longer elements, and as the frequency increases, the active cell moves toward the shorter elements. With variations of the design constant, τ, the apex half angle, α (or relative spacing constant σ), and the element-to-element feeder spacing, S, the following trends were found:

1) The gain increases as τ increases (more elements for a given f) and α decreases (wider element spacing).

2) The average input impedance decreases with increasing α (smaller element spacing) and increasing τ (more elements for a given f).

3) The average input impedance decreases with decreasing S, and increasing conductor size of the element-to-element feeder.

The LPV array operates at higher order resonance points, as described earlier. That is, energy is readily accepted from the feeder by those elements which are near any of the odd-multiple resonances ($\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.). The higher order modes of the LPV array are higher order space harmonics[6] and hence, when an

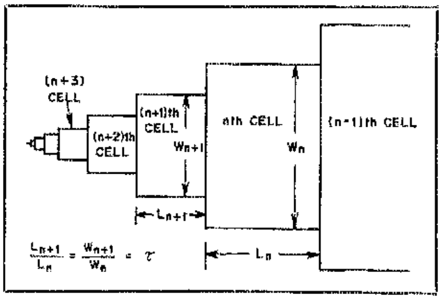


Fig. 4 — An interconnection of a geometric progression of cells.

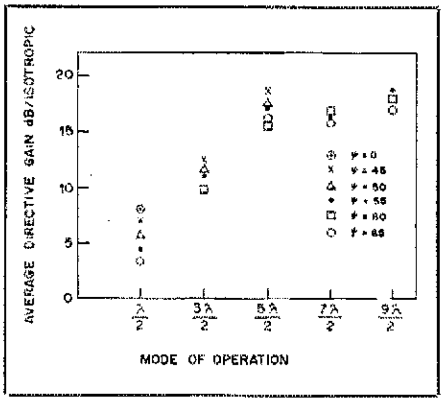


Fig. 5 — Average directive gain above isotropic (dBi). Subtract 2.1 from gain values to obtain gain above a dipole (dBd).

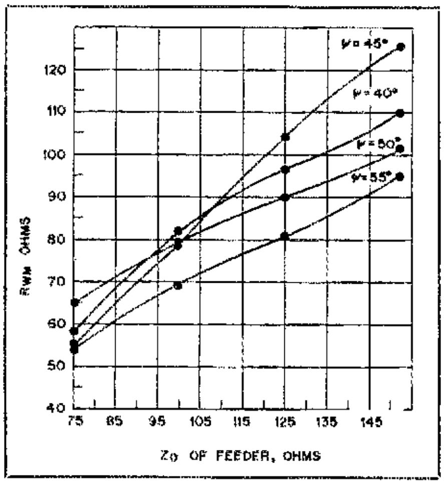


Fig. 6 — Weighted mean resistance level, R_{wm}, vs characteristic impedance of the feeder Z_0 for various ψ angles.

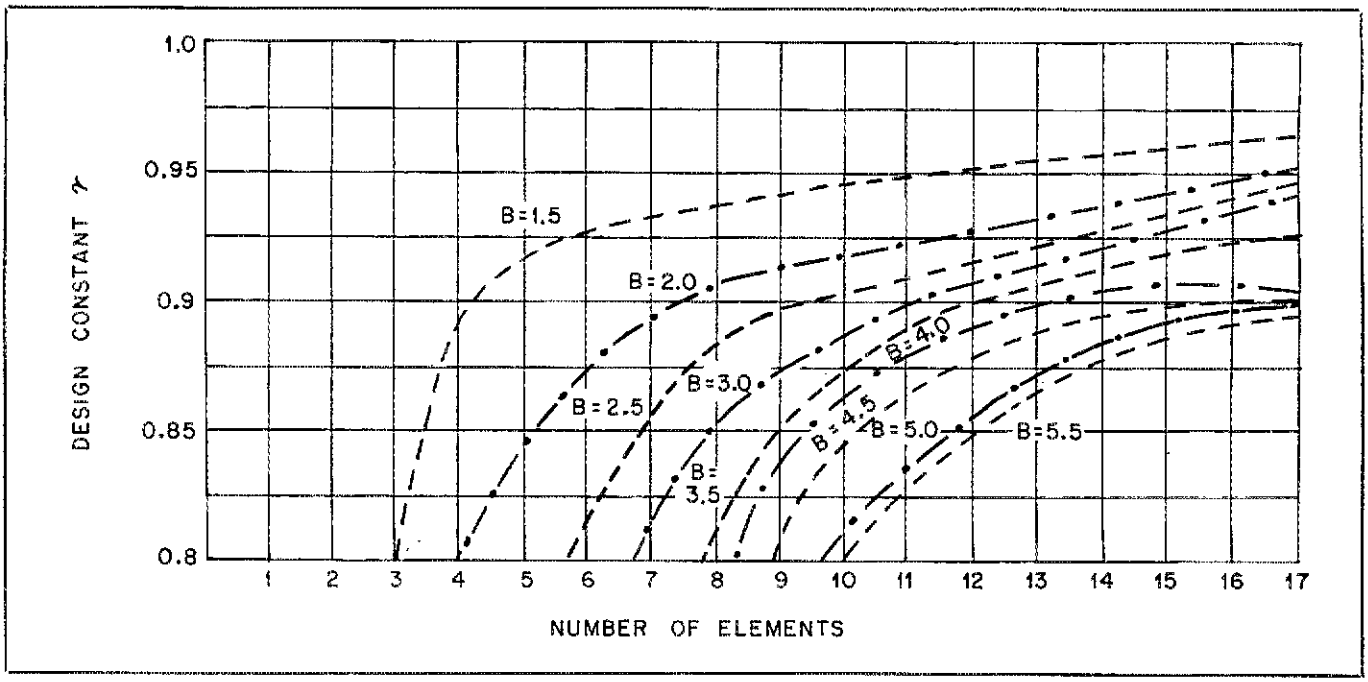


Fig. 7 — Design parameter, τ, vs number of elements, N, for various operational bandwidths, B.

LPV array is operated at a half-wavelength frequency shorter than the smallest element, the energy on the feeder will propagate to the vicinity of the $3\lambda/2$ element and be radiated. (See ref. 4.)

The elements are tilted toward the apex of the array by an angle, ψ, shown in Fig. 1. The tilt angle, ψ, determines the radiation pattern and subsequent gain in the various modes. For each mode there is a different tilt angle which produces maximum gain.[7] Mayes and Carrel did extensive experimental work with an LPV of 25 elements with $\tau = 0.95$ and $\sigma = 0.0268$. The tilt angle, ψ, was varied from 0° to 65° and radiation patterns plotted in the $\lambda/2$ through $7\lambda/2$ modes. The gain curves appear in Fig. 5. The E- and H-plane patterns are found in Fig. 2. Operation in the higher modes is improved by increasing τ (more elements) and decreasing σ (closer element spacing).

$$\sigma = 1/4\,(1 - \tau)\cot\alpha \qquad \text{(Eq. 4)}$$

When considering any single mode, the characteristic impedance is comparable with that of the LPD array; it is predominantly real and clustered around a central value, R_o. The central value R_o for each mode increases with Z_o (feeder impedance). Thus, control of the input impedance can be accomplished by controlling Z_o.

When multimode operation is desired, a compromise must be made in order to determine a fixed impedance level. The multimode array impedance is defined as the weighted mean resistance level, R_{wm}. Also, it can be shown that R_{wm} lies between the R_o central values of two adjacent modes. (See ref. 4.) For example:

$$R_{o1/2} < R_{wm} < R_{o3/2} \qquad \text{(Eq. 5)}$$

where

$R_{o1/2}$ = $\lambda/2$ mode impedance, center value

$R_{o3/2}$ = $3\lambda/2$ mode impedance, center value

and:

$$R_o = \sqrt{R_{max} \bullet R_{min}} \qquad \text{(Eq. 6)}$$

$$VSWR = \sqrt{\frac{R_{max}}{R_{min}}} \qquad \text{(Eq. 7)}$$

The weighted mean resistance level between the $\lambda/2$ and $3\lambda/2$ modes is defined by

$$R_{wm} = \sqrt{R_{o1/2}\,R_{o3/2}\,\frac{VSWR_{3/2}}{VSWR_{1/2}}} \qquad \text{(Eq. 8)}$$

where

$VSWR_{1/2}$ = VSWR in $\lambda/2$ mode

$VSWR_{3/2}$ = VSWR in $3\lambda/2$ mode

Once Z_o and ψ have been chosen, Fig. 6 can be used to estimate the R_{wm} value for a given LPV array. Notice the dominant role Z_o (feeder impedance) plays in the array impedance.

It is apparent from the preceding data that the LPV is useful for covering a number of different bands spread over a wide range of the spectrum. It is fortunate that most of the amateur bands are harmonically related, and by choosing a large design parameter, $\tau = 0.9$, a small relative spacing constant, $\sigma = 0.02$, and a tilt angle of $\psi = 40°$, and LPV could easily cover the amateur bands from 40 through 6 meters!

Design Procedure

1) Determine operational bandwidth, B, in $\lambda/2$ mode:

$$B = \frac{f_n}{f_1} \qquad \text{(Eq. 9)}$$

where

f_n = highest freq. in MHz

f_1 = lowest freq. in MHz

2) Determine τ for a desired number of elements, n, using Fig. 7.

3) Determine element lengths ℓ_1 to ℓ_n using Eqs. 1 and 2.

4) Choose the highest operating mode desired and determine σ and ψ from Fig. 8.

5) Determine cell boom length, L.

$$L = \frac{2\sigma(\ell_1 - \ell_n)}{(1 - \tau)} \qquad \text{(Eq. 10)}$$

Note: If more than one LPV cell is to be driven by a common feeder, the spacing between cells can be determined by:

$$D_{1-2} = 2\sigma_1 \ell_{n1}$$

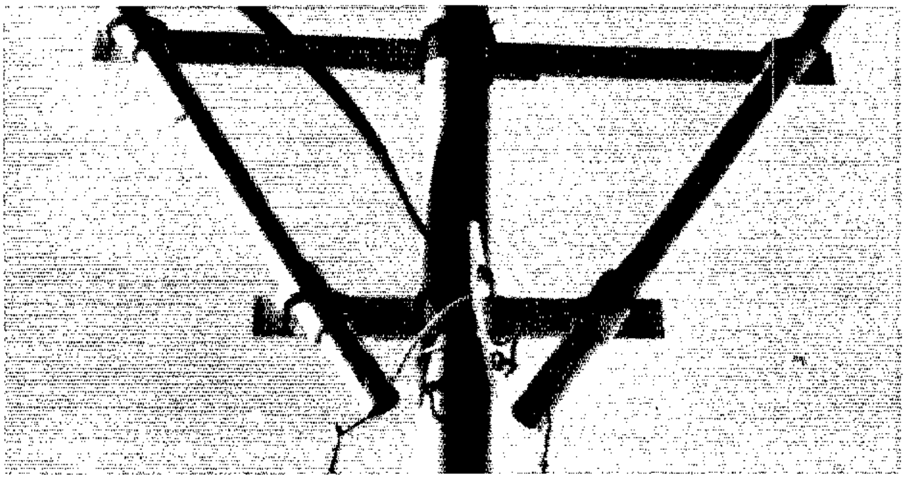
where

D_{1-2} = element spacing between cell 1 (lower frequency cell) and cell 2 (higher frequency cell).

σ_1 = relative spacing constant for cell 1

ℓ_{n1} = shortest or last element within cell 1

6) Determine the mean resistance level, R_{wm}, using Fig. 6.



The element-to-boom detail is depicted here. Aluminum angle brackets, U bolts, and sections of PVC tubing are shown securing each element to the boom at two points. The 300-ohm twin-lead, threaded through a piece of polystyrene and attached to the foremost element, may be seen entering the picture at the top/left. The end of linear loading line, ℓ_1, is visible near the bottom.

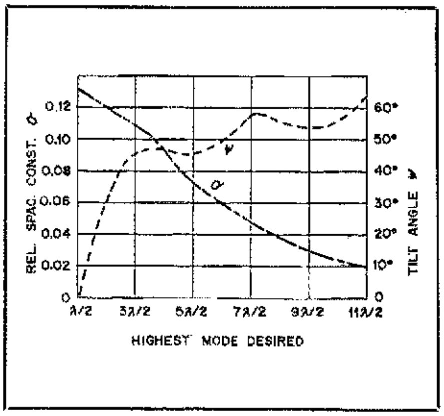


Fig. 8 — Optimum σ and ψ for an LPV when the highest operating mode has been chosen.

7) Determine the element spacings using equations 3 and 4.
This completes the design.

Construction Considerations

The 7- to 30-MHz LPV in use at K4EWG has given good results and construction data are warranted. However, exhaustive detailed drawings and material lists are omitted since it is my intent to stimulate interest, rather than produce the last word in amateur-band LPV electrical and structural design.

It may be of interest to note that both linear and capacitive loading were used on ℓ_1. The relationship in the appendix may be used in estimating linear loading-stub length and/or capacitive hat size if construction constraints prohibit a full-sized array. However, performance in higher mode operations was less than optimum when shortened elements were used.

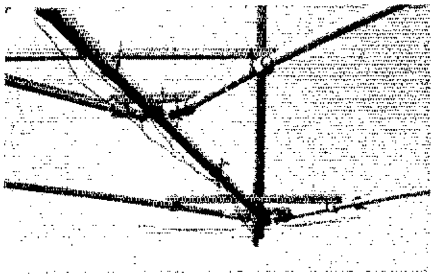
The structural details can be seen in the photos and more specific data can be found in Table 1. Array performance patterns can be found in Fig. 3. The array was fed directly with 300-ohm twin-lead.

Summary

The LPV provides frequency-independent coverage of each of several frequency bands. In higher mode operation, 2 to 3 dB of additional gain can be obtained from the same physical structure without degrading the pattern or characteristics of the array. It is my hope that this article will stimulate additional research by the amateur fraternity.

Appendix

The following linear loading-stub design equation may be used for approximating the stub length (one half of element, two stubs required):



A shot of the rearmost element looking at an angle to the boom. The linear loading line may be seen supported at various points along the boom and at the rear element by pieces of polystyrene.

$$L_s = \frac{2.734}{f} \arctan \left[\frac{33.9\,[\ell_n \frac{24h}{d} - 1]\,[1 - (\frac{fh}{234})^2]}{fh \,\mathrm{Log}\,(\frac{b}{a})} \right]$$

where

L_s = linear loading-stub length in feet required for each half element
h = element half length in feet
f = element resonant frequency in MHz
b = loading stub spacing in inches
a = radius of loading stub conductors in inches
D = average element dia in inches

Note: The resonant frequency, f, of an individual element of length, ℓ, can be found using:

$$f = \frac{467}{\ell}$$

The capacitive hat dimensions for each half element can be found using the excellent data by W. Schulz, K3OQF.[8] QST

Table 1
Design Dimensions

Element Lengths, ft	*Element Spacings, ft*	*Design Parameters*
ℓ_1 = 56.22	d_{12} = 9.15	τ = 0.8
ℓ_2 = 56.22	d_{23} = 7.32	σ = 0.05
ℓ_3 = 45.0	d_{34} = 5.86	α = 38.2°
ℓ_4 = 36.0	d_{45} = 4.67	**L = 27'
ℓ_5 = 28.79		ψ = 45°

Feet (') × 0.3048 = m
Inches (") × 25.4 = mm

*ℓ_1 is a shortened element; the full size dimension is 70.28 ft.
**The total physical boom length is L plus the distance to the ℓ_5 cross bracing. The cross-braces are 3 ft. in length and ψ = 45°; hence, the total boom length is 27 ft + 1.5 ft = 28.5 ft.

Table 2
Basic Materials

Elements	1-1/2", 6061-T6, 0.047" wall aluminum tubing
Bracing	1-1/4" × 1-1/4" × 1/8" aluminum angle
Boom	2-1/2" OD, 0.107" wall aluminum tubing
U bolts	1/4" squared at loop to accommodate tilt angle, ψ
Feeder	no. 12, solid copper.
Cap. hat. for ℓ_1	no. 10 alum. wire, 24" dia
Linear loading for ℓ_1	4' loop, 3" spacing each half of ℓ_1.

References

[1]Rhodes, "The Log-Periodic Dipole Array," *QST*, November 1973.
[2]"The Log-Periodic Dipole Array," *The ARRL Antenna Book*, 1974, pp. 160-164.
[3]Rhodes and Painter, "The Log-Yag Array," *QST*, December 1976.
[4]Mayes and Carrel, "Log Periodic Resonant-V Arrays," I.R.E., Wescon Convention Record, Part 1, 1961.
[5]Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, NY, 1941, p. 488.
[6]Mayes, Deschamps and Patton, "Backward Wave Radiation from Periodic Structures and Application to the Design of Frequency Independent Antennas," Proceedings of I.R.E., vol. 49, no. 5, May 1961.
[7]Kraus, *Antennas*, McGraw-Hill, New York, NY, 1950, p. 25.
[8]Schulz, "Designing a Vertical Antenna," *QST*, September 1978.